A Unified Uncertainty based Approach for Optimal Quality Decisions

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Problem: Confusion

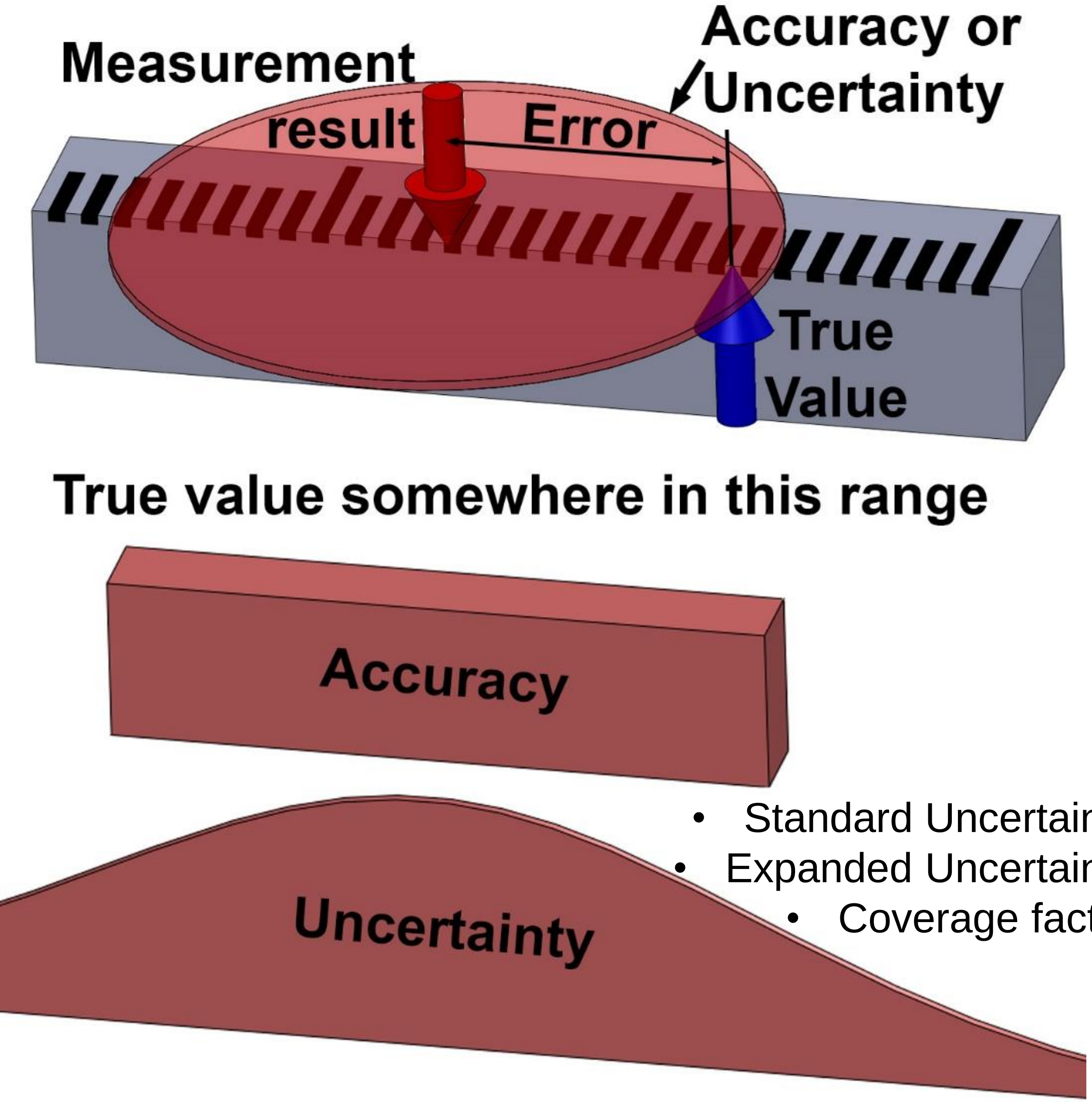
For quality we must understand uncertainty

But we don't understand how to apply uncertainty to manufacturing… and GUM hasn't been adopted

Problem

- Different methods and terms for equivalent quantities
- No single system is fit for purpose
- Arbitrary targets like 'Six-Sigma' are not optimal

Accuracy and Uncertainty

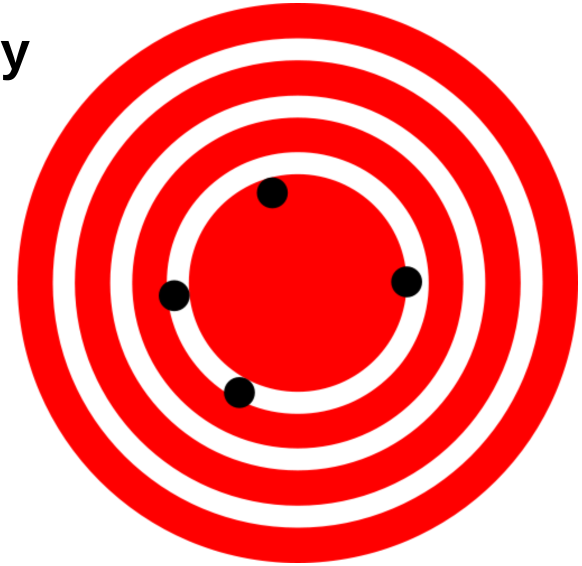


- Standard Uncertainty
- Expanded Uncertainty
- Coverage factor

Similar terms:

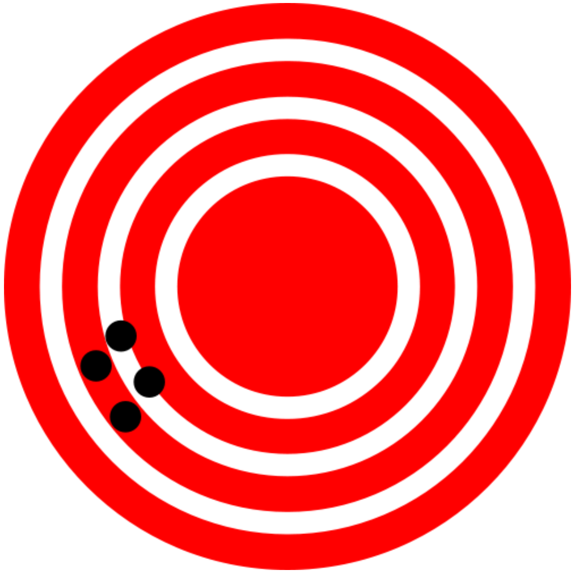
- Error source: **MSA**
- Influence quantity: **Uncertainty**
- Factor: **SPC**

Random & Systematic Effects



- Random effects / Random uncertainty: **Uncertainty**
- Random error / Precision: **MSA**
- Common causes: **SPC** (was chance causes)

Repeatability (Short-term variability in SPC)	**Reproducibility** (Long-term variability in SPC)



- Bias & Trueness: **MSA**
- Systematic effects: **Uncertainty**
  - Caused by *Influence quantities*
- Special cause variation: **SPC**
  - Was assignable cause
  - When all compensated, so negligible special cause variation, process is in *Statistical Control:* ***SPC*** *(was* Stable process)

No single system is fit for purpose

MSA

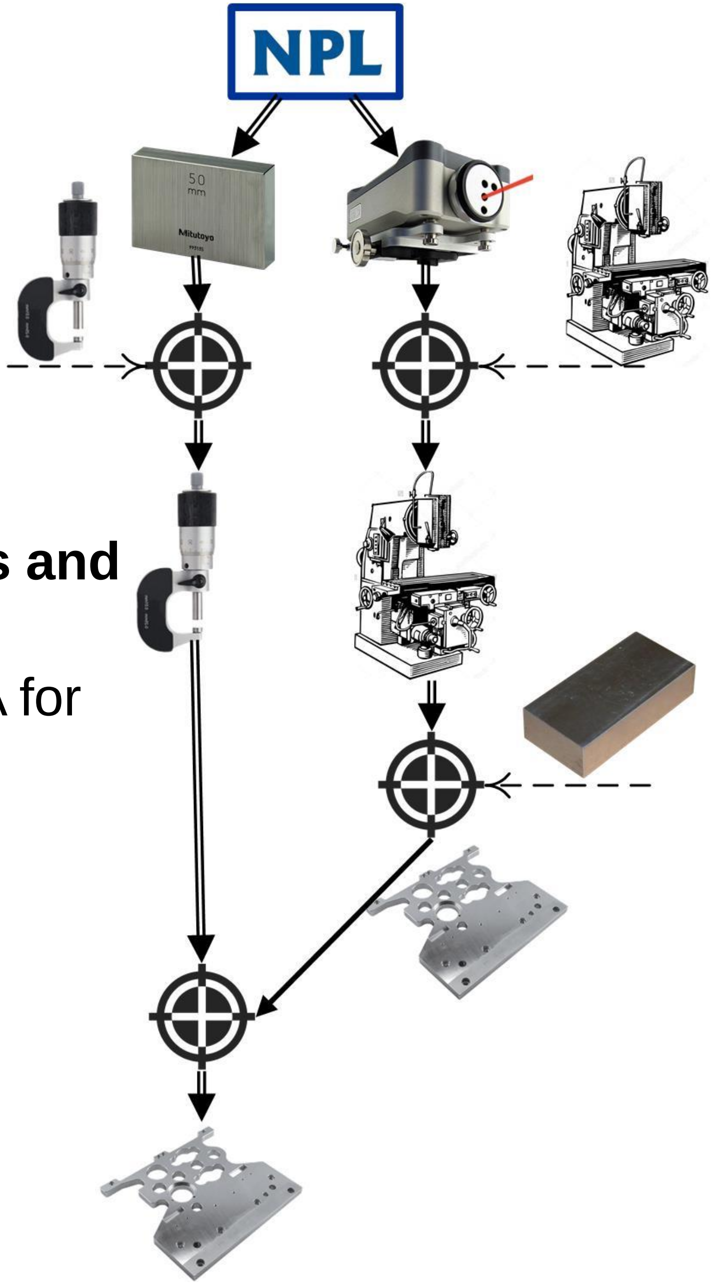
- 📍 Factory measurements
- ☹ **Gauge studies often underestimate reproducibility conditions**
- ☹ Ignores some systematic effects

SPC

- 📍 Factory processes
- ☹ **Ignores covariance between process and measurement, may hide errors!**
- ☹ May overestimate variation due to MSA for measurements
- ☹ Tests for ‘in-control’ differ in sensitivity

GUM Uncertainty

- 📍 Calibration labs
- ☹ **After-the-fact only**
- ☹ **Assumptions often not valid**
- ☹ Needs mathematical model



Gage studies & reproducibility

- Gage R&R study often seen as ‘truth’ in industry
- But reproducibility often limited to part and operator!
- Environmental and material property variations often not represented
- GUM approach forces us to consider effect of all influences

SPC can miss systematic effects

- Consider a steel gage measuring a part produced on a steel machine
- Temperature varies
- Expansion of machine and gage cancel
- Significant variation may not appear in SPC data

$$L_{M(T+\Delta T)} = L + \Delta T\, L\, \alpha_M$$

$$L_{P(T)} = L_{M(T+\Delta T)} - \Delta T\, L\, \alpha_P$$

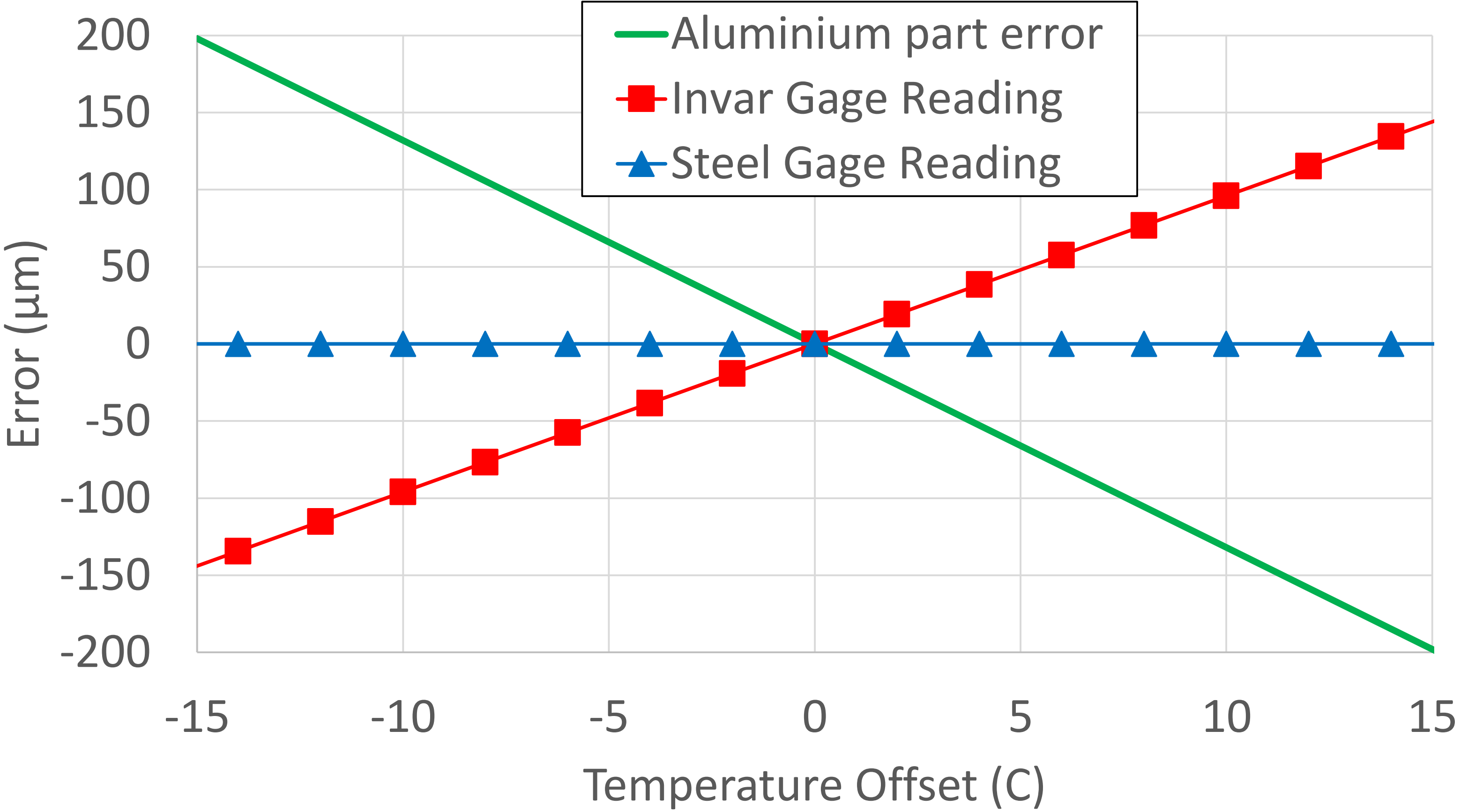
$$\boxed{L_{P(T)} = L + \Delta T\, L\, (\alpha_M - \alpha_P)}$$

$$L_G = L_{M(T+\Delta T)} - \Delta T\, L\, \alpha_G$$

$$\boxed{L_G = L + \Delta T\, L\, (\alpha_M - \alpha_G)}$$

SPC can miss systematic effects

Part produced on steel machine tool:



An uncertainty evaluation (GUM) approach would identify that the gage is not capable, but normally MSA is used which can easily miss this effect.

Limitations of GUM approach

- GUM is ‘after-the-fact’ i.e. correction values must already be known to evaluate uncertainty
- GUM assumes Gaussian output which is only exact for linear models

- I will use correction for thermal expansion to gives examples of these issues

$$\Delta L = \propto \Delta T \, L_0$$

- Linear assumption is valid
- Typical uncertainties (95%)
  - $\propto$: 6% to 10%
  - ΔT : 0.1 °C to 0.5 °C
  - L_0 : Typically negligible
- Often significant and sometimes dominant

GUM is after-the-fact

$$\Delta L = \propto \Delta T\, L_0$$

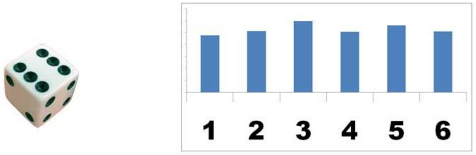
$$u_{\Delta L}^2 \approx (\alpha\, \Delta T\, u_L)^2 + \ (L\, \Delta T\, u_\alpha)^2 + (L\, \alpha\, u_T)^2$$

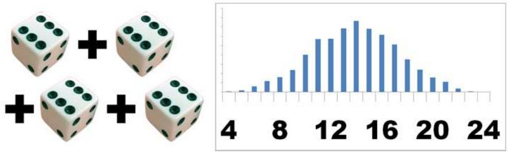
- GUM assumes each input quantity has been determined
- We often need uncertainty before they are determined
  - Estimate uncertainty for a planned measurement
  - Determine probability of parts conforming
- Two approaches typically used
  - If uncertainty in the input has negligible effect use nominal value
  - If it is significant use worst case value
- Why use worst case?
  - Because GUM doesn’t have a solution!
- Modelling this is easy if we consider errors and propagate the uncertainty in these errors with MCS

Temp. Correction: GUM Approach

$$\Delta L = \propto \Delta T\, L_0$$

$$u_{TE}^2 \approx (\alpha\, \Delta T\, u_L)^2 + (L\, \Delta T\, u_\alpha)^2 + (L\, \alpha\, u_T)^2 + (\Delta T\, u_L\, u_\alpha)^2 + (\alpha\, u_L\, u_T)^2 + (\mathrm{L}\, u_\alpha\, u_T)^2 + (u_T\, u_L\, u_\alpha)^2$$





- Terms evaluated for 14400 combinations of parameters:
  - Lengths between 1 µm and 100 m
  - Fractional standard uncertainty in length of between 10^{-7} and 10^{-3}
  - CTE's between 1.2 and 23 ppm/°C
  - Fractional standard uncertainty in CTE of between 0.2% and 37%
  - Temperature offsets of between 0.01 °C and 20 °C
  - Fractional standard uncertainty in measurement of the temperature offset of between 0.001 °C and 2 °C.

Scale Correction: GUM Approach

$$\Delta L = \propto \Delta T\ L_0$$

$$\begin{aligned} u_{TE}^2 \approx {} & (\alpha\ \Delta T\ u_L)^2 + (L\ \Delta T\ u_\alpha)^2 + (L\ \alpha\ u_T)^2 \\ & + \cancel{(\Delta T\ u_L\ u_\alpha)^2} + \cancel{(\alpha\ u_L\ u_T)^2} + (\mathrm{L}\ u_\alpha\ u_T)^2 \\ & + \cancel{(u_T\ u_L\ u_\alpha)^2} \end{aligned}$$

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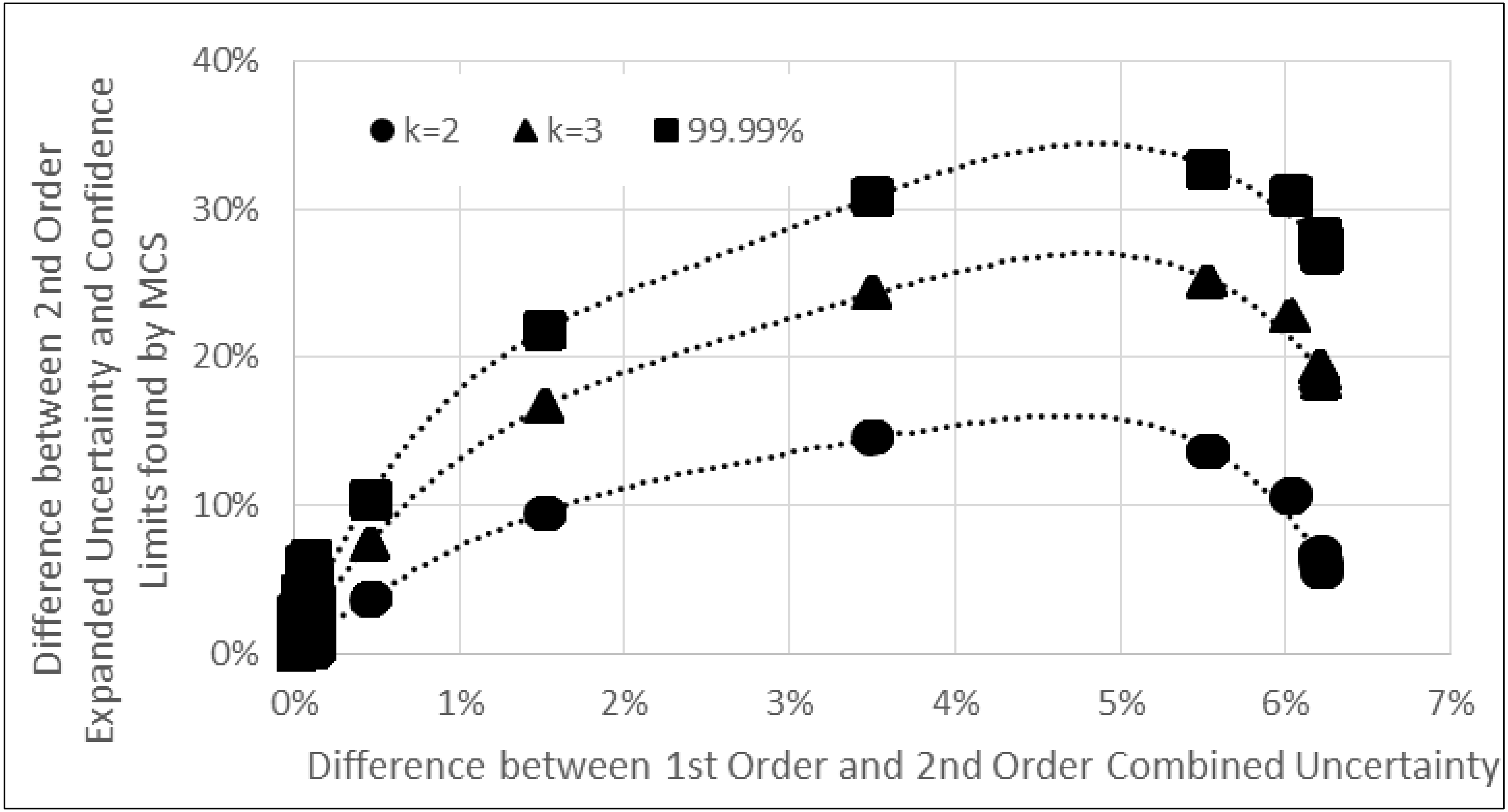
Monte Carlo Verification (GUM-S1)



Optimize for Cost per Part Sold

A part meets 1 of 4 conditions, with finite probabilities:

	Pass Verification	Fail Verification
In-Spec'	True Negative, P_1	False Positive, P_3
Out-of-Spec'	False Negative, P_4	True Positive, P_2

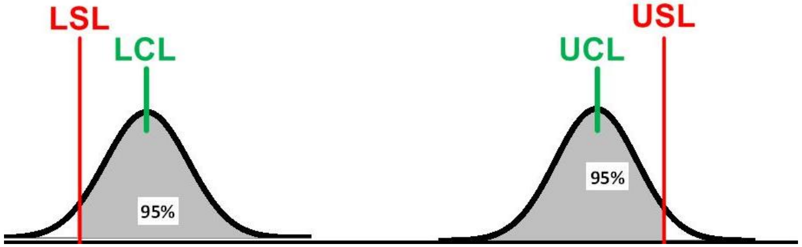
- Cost of manufacture (C_1) occurs for every part
- Defects reaching customer have additional cost (C_2)

$$C_Q = \frac{C_1 + P_4\, C_2}{P_1 + P_4}$$

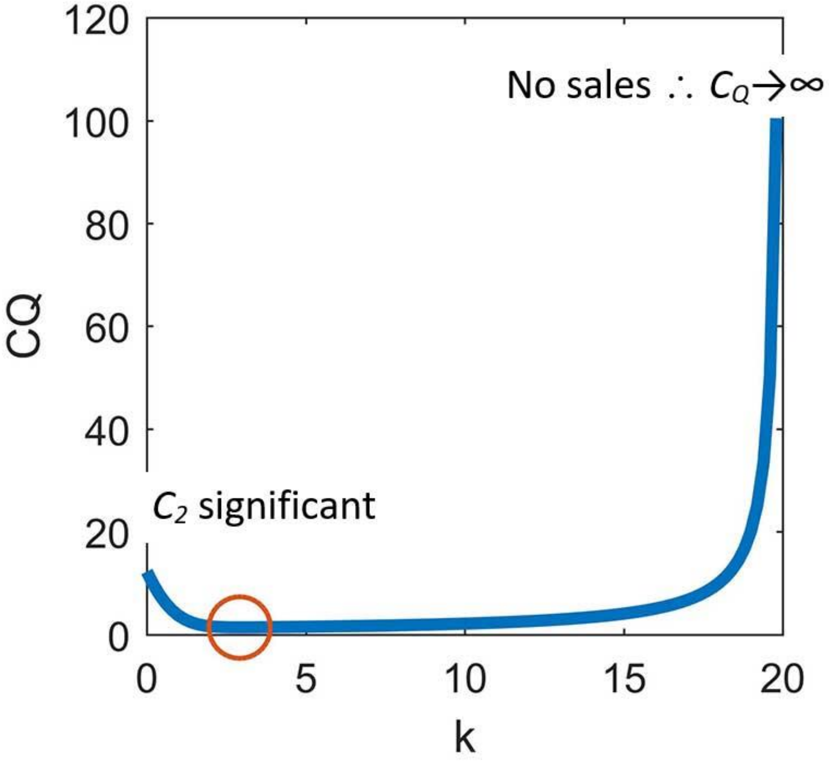
$M = U/T$

$P = \sigma/T$

k

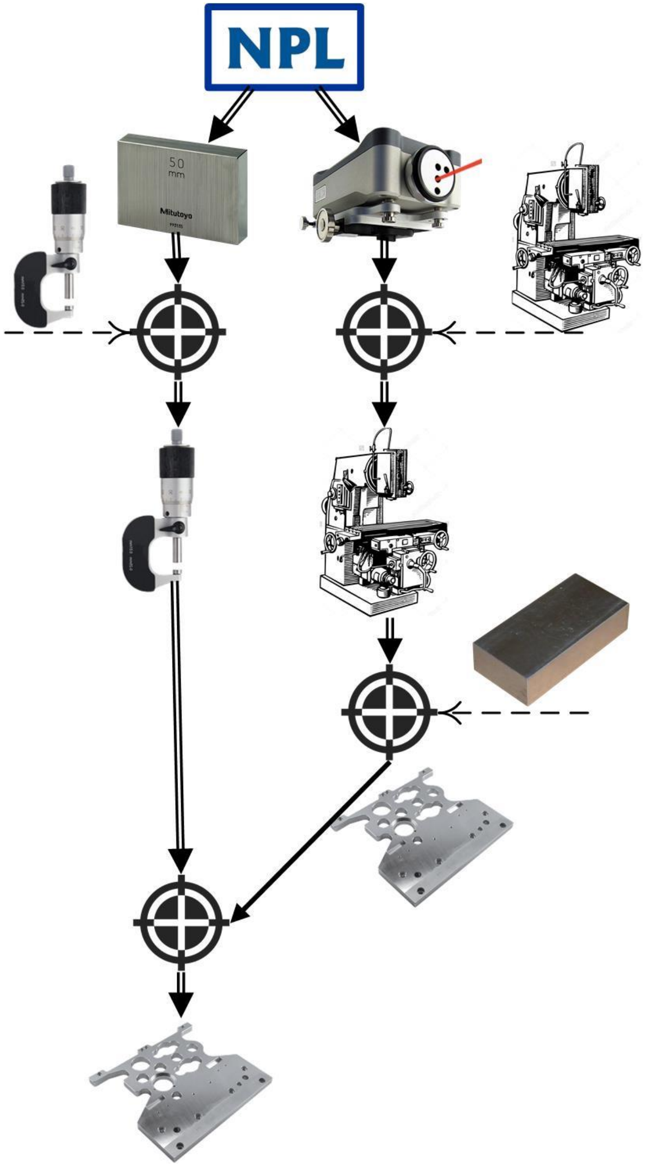


1. *M* and *P* discreet values (instruments and machines). Try all combo's
2. Find *k* to minimize C_Q for each combination (PS using MVN CDF)



Unified Uncertainty

- Cost based optimization algorithms
  - Process selection
  - Instrument selection
  - Set of conformance limits
- Standardised terminology
- Uncertainty evaluation algorithms
  - Generic models of influences
  - Before-the-fact uncertainty
  - Non-Gaussian distributions
- Algorithms for experimentally verifying uncertainty models



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