Uncertainties in Dimensional Measurements due to Thermal Expansion

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Abstract

Thermal expansion is a source of uncertainty in dimensional measurements, which is often significant and in some cases dominant. Methods of evaluating and reducing this uncertainty are therefore of fundamental importance to product quality, safety and efficiency in many areas. Existing methods depend on the implicit assumption that thermal expansion is relatively uniform throughout the part and can therefore be corrected by scaling the measurement result. The uncertainty of this scale correction is then included in the uncertainty of the dimensional measurement. It is shown here that this assumption is not always valid due to thermal gradients resulting in significant shape changes. In some cases these are the dominant source of dimensional uncertainty. Methods are described to first determine whether shape change is significant. Where shape changes are negligible but thermal expansion remains significant then the established methods may be used. This paper describes the application of the Guide to the expression of Uncertainty in Measurement (GUM) uncertainty framework which provides an approximate solution for thermal expansion due to non-linearity and a non-Gaussian output function. The uncertainty associated with this approximation is rigorously evaluated by comparison with Monte Carlo Simulation over a wide range of parameter values. It is often necessary to estimate the expected uncertainty for a measurement which will be made in the future. It is shown that the current method for this is inadequate and an improved method is given.

1 Introduction

Part dimensions vary with temperature; typically expanding with increasing temperature. The rate for a specific material is given by its coefficient of thermal expansion (CTE). Typical values are between 1.2 and 23 parts per million (ppm) per degree of temperature increase. For normal environmental temperature ranges a linear rate of expansion can be assumed so that the change in length is approximated by

$$\Delta L = \propto \Delta T \; L_0 \tag{1}$$

where α is the coefficient of thermal expansion, ΔT is the change in temperature and L_0 is the initial length.

Product dimensions are specified at a standard reference temperature of 20°C. Measurement at a different temperature will result in an error. The part temperature should therefore be measured in order to correct for this error. Part temperature is normally assumed to be uniform so that thermal expansion does not result in shape changes. If this assumption is valid the correction can be made using Eqn. (1). Uncertainty arises from the uncertainties in the determination of the part's temperature, length and CTE. A review of the uncertainties in dimensional measurements due to temperature variation was carried out in 1994 by Swyt [1], which showed that for measurements carried out with state of the art measurement control the uncertainty due to thermal expansion remained a significant but not a dominant source of uncertainty ranging from 14% to 25% of the combined uncertainty of measurement. For more typical measurements temperature can have a much greater effect.

Recent reviews of temperature sensor accuracy [2, 3] give expanded uncertainties, at 95% confidence, as from 0.01°C for state of the art Industrial platinum resistance thermometers (IPRTs) [4] and Thermistors [5], to between 0.1 °C and 0.5 °C for low cost Thermocouples [6]. In practice however few industrial measurements are better than 0.1 °C. A state of the art CMM calibration facility may achieve an expanded uncertainty in temperature control of 0.05°C compared with 0.5°C for most metrology laboratories [7]. Many industrial measurements are made within uncontrolled environments where an expanded uncertainty in the single point temperature of 5°C and spatial temperature gradients of 1°C/m would be typical [8]. Expanded uncertainty of part CTE may be as high as 100% for broad classes of material such as carbon steel or aluminium alloy, reducing to approximately 10% for a known grade such as gauge quality carbon steel. With known chemical composition it reduces further to between 6% and 9%, still much higher than the 0.3% possible by measuring samples with dilatometry [1].

2 Uncertainty Evaluation for Scale Correction

The Guide to the Expression of Uncertainty in Measurement (GUM) [9] gives analytical methods to combine uncertainty. These are exact when the measurement result is the linear combination of a number of input quantities, each having independent, random and normally distributed uncertainty. The method is summarized by

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) \qquad (2)$$

where $u_c^2(y)$ is the variance representing the combined standard uncertainty of the measurement result y, $\partial f / \partial x_i$ is the sensitivity of the measurement result y to input quantity x_i and $u(x_i)$ is the standard uncertainty in x_i.

For cases, such as thermal expansion, where the measurement result is a non-linear function of a number of input quantities Eqn. (2) may not properly represent the combination of uncertainty. In such cases the GUM recommends that higher order terms are included. However, this is still an approximate solution. It depends on assumptions including the Central Limit Theorem. The first supplement to the GUM notes that quantifying the effects of these approximations is difficult. It recommends that Monte Carlo simulation (MCS) is used to validate the analytical equations [10].

A scale correction for thermal expansion involves determining the quantities ΔT, L and α, with some uncertainty, and applying Eqn. (1). Following the GUM uncertainty framework and including all non-zero terms in the Taylor-series expansion, the uncertainty due to thermal expansion is given by:

$$\begin{aligned} u_{TE}^2 \approx\ & (\alpha\, \Delta T\, u_L)^2 + (L\, \Delta T\, u_\alpha)^2 + (L\, \alpha\, u_T)^2 \\ & + (\Delta T\, u_L\, u_\alpha)^2 + (\alpha\, u_L\, u_T)^2 + (\mathrm{L}\, u_\alpha\, u_T)^2 \\ & + (u_T\, u_L\, u_\alpha)^2 \end{aligned} \qquad (3)$$

where uL, u_α and u_T are the uncertainties in the part dimension, CTE and the temperature offset respectively. The first order terms are given on the first line, second order on the second line and third order on the third line.

A generalized validation of this equation against MCS was carried out. All terms of Eqn. (3) were evaluated over 14400 combinations of parameters. This covered lengths between 1 µm and 100 m, fractional standard uncertainty in length of between 10^{-7} and 10^{-3}, CTE's between 1.2 nd 23 ppm/°C, fractional standard uncertainty in CTE of between 0.2% and 37%, temperature offsets of between 0.01 °C and 20 °C and fractional standard uncertainty in measurement of the temperature offset of between 0.001 °C and 2 °C. For each of these evaluations comparison was also made to simulation with 10^8 Monte Carlo trials which was shown by convergence study to provide a standard deviation of the simulation results of less than 1% for confidence limits up to 99.999% with considerably better accuracy for lower confidence limits and for the standard uncertainty. The comparison showed that, under the range of conditions simulated:

1. The first order term involving uL, which is sometimes ignored, is never dominant but can sometimes contribute approximately 20% of the total variance. Both of the other first order terms can be dominant.
2. Only one of the higher order terms is significant; the $u\alpha \cdot uT$ term can contribute up to 12% of total variance. None of the other higher order terms ever contribute more than 0.01% of the total variance.
3. Where the higher order term is significant this results in a non-Gaussian output distribution meaning that errors in the confidence limits are considerably higher than errors that would have been seen in the standard uncertainty if a first order approximation had been used. The higher the confidence level the larger these errors become. This effect makes the use of higher order approximations to estimate uncertainty highly questionable as illustrated in Figure 1.

Figure 1: Effect of Significant Higher Order Terms on Confidence Limits

The uncertainty of measurement is intended to characterize the dispersion of the values that could reasonably be attributed to the measurand. The GUM method uses the assumption of normality to infer this from a standard deviation. Applying second order terms when this assumption is not valid does not make sense. Second order terms should therefore not be used to find the standard uncertainty. The difference between the first order and second order approximation should be used to determine whether the normality assumption is valid, where it is valid the analytical approach may be taken. Where the normality assumption is not valid then confidence limits should be calculated directly from MCS.

3 Uncertainty Evaluation for a Planned Correction for Thermal Expansion

When estimating the uncertainty of a planned measurement there are two approaches which are typically used to estimate expected values for input quantities a) where likely deviations in the nominal value of an input quantity would result in negligible changes in the combined uncertainty then the nominal value is used; b) where different likely values for the input quantity would result in significantly different values for the combined uncertainty then a worst case value is used [11]. The CTE will normally be known in advance and the measured length can be assumed to be the nominal length since small changes in the length have a negligible effect on the uncertainty of the correction for thermal expansion. The measured temperature offset, however, is likely to vary considerably with a significant effect on the resulting uncertainty. A worst case value would therefore normally be used for the temperature offset. This is, however, inconsistent since other uncertainties are not combined using worst case values. A consistent approach would be to combine the ***uncertainty of the temperature offset ($U_{\Delta T}$)*** with the ***uncertainty of the measurement of this offset (U_T)***. The GUM uncertainty framework does not provide a method with which to combine an uncertainty in the expected value of an input quantity with an uncertainty in its determination and this type of bivariate normal distribution must be found by a numerical method.

The error in a correction for thermal expansion is the difference between the true value of the thermal expansion and the correction made for it. The true value of the thermal expansion cannot be determined, but is given theoretically by

$$T_{TE} = L\, E_{\Delta T}\, \alpha \tag{4}$$

where L is the true length at the reference temperature, α is the true value of the CTE and $E_{\Delta T}$ is the temperature offset at which measurement is carried out.

$E_{\Delta T}$ is considered as an error in this case since the temperature control attempts to maintain a zero temperature offset. The value of the correction for thermal expansion is given by

$$M_{TE} = (L + E_L)(\mathrm{E}_{\Delta T} + E_T)(\alpha + E_\alpha) \tag{5}$$

where E_L is the error in the length measurement, E_T is the error in the measurement of the part's temperature and E_α is the error in the determination of the part's CTE.

The error in the correction for thermal expansion is simply the measured thermal expansion minus the true thermal expansion, given by

$$E_{TE} = M_{TE} - T_{TE} \tag{6}$$

The errors $E_{\Delta T}$, E_L, E_T and E_α are unknown quantities each having an expectation of zero and a dispersion of values that could reasonably be attributed to them characterized by their uncertainties; $U_{\Delta T}$, U_L, U_T and U_α respectively. $U_{\Delta T}$ is determined by the stability and accuracy of temperature control in the measurement environment and is assumed to have a rectangular distribution. The other uncertainties are each the uncertainty with which the corresponding input quantity is determined during the measurement process, they are assumed to have normal distributions.

MCS was used to simulate the error E_{TE} by generating random values for the errors $E_{\Delta T}$, E_L, E_T and E_α drawn from the probability density functions characterizing $U_{\Delta T}$, U_L, U_T and U_α respectively. The results of simulation using 10^8 Monte Carlo trials were compared with the analytical solution using Eqn. (3). For the analytical solution both the worst case value for ΔT and the standard uncertainty in ΔT were used. The solutions were compared both in terms of calculated standard uncertainty and confidence limits. This comparison was made for 14400 combinations of parameters similar to those used in the previous section.

It was found that using the standard uncertainty in the temperature offset (the half-limit of the rectangular distribution divided by $\sqrt{3}$) Eqn. (3) gave a close agreement with the MCS for the combined standard uncertainties. The agreement was within 0.05% for all measurement scenarios. Using the temperature offset as a worst case value (the half-limit of the distribution) resulted in differences of up to 73%. However, even using the uncertainty in the temperature offset, considerably larger differences were seen for confidence limits with differences of up to 11% for k=2 and 24% for k=3, where *k* is the confidence level. In this case the reliability of confidence limits cannot be estimated by considering the contribution from higher order terms since the first order terms also produce non-Gaussian output distributions. It is therefore extremely difficult to predict the accuracy of the analytical solution and MCS should be used whenever possible.

4 Effects of Spatial Temperature Variation

The above analysis assumed that any difference between the part temperature and the reference temperature can be represented by a single mean temperature offset. In many cases this may be true. There must, however, always be some consideration of the spatial temperature variation since the average temperature of the part cannot be assumed to be a single point temperature measurement with uncertainty equal to the sensor uncertainty. The spatial temperature variation should therefore be evaluated as a contribution to the part temperature uncertainty. A second cause of uncertainty due to spatial temperature variation is distortion of part shape due to temperature gradients. This is an entirely different problem to the scale changes produced by linear expansion and one which has been largely ignored in previous research [12].

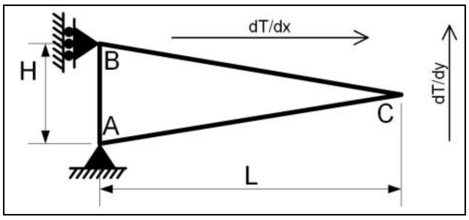

Figure 2: Example of Thermal Gradients Causing Shape Changes in a Truss

Consider a two dimensional L by H truss with primary datum at point A, secondary datum at point B and some other point to be measured at C. The structure is to be measured in the presence of temperature gradients *dT/dx* and *dT/dy* as shown in Figure 2. Assuming the members are in equilibrium with the environment, then the average temperature offsets are given by

$$\Delta T_{AB} = T_A - T_{ref} + \frac{H}{2}\frac{dT}{dy} \tag{7}$$

$$\Delta T_{AC} = T_A - T_{ref} + \frac{H}{4}\frac{dT}{dy} + \frac{L}{2}\frac{dT}{dx} \tag{8}$$

$$\Delta T_{BC} = T_A - T_{ref} + \frac{3H}{4}\frac{dT}{dy} + \frac{L}{2}\frac{dT}{dx} \tag{9}$$

where ΔT_{AB}, ΔT_{AC} and ΔT_{BC} are the average temperature offsets of members AB, AC and BC respectively, T_A is the temperature at point A and T_{ref} is the reference temperature, assumed to be 20°C.

Assuming that thermal expansion produces a uniform scaling effect then the thermal expansion of point C would be given by

$$\Delta C_{x(scaling)} = \frac{\Delta T_{AC} + \Delta T_{BC}}{2}\alpha L \tag{10}$$

$$\Delta C_{y(scaling)} = \Delta T_{AB}\alpha\frac{H}{2} \tag{11}$$

Considering the actual member's lengths, with thermal expansion, these become

$$L_{AB} = H(1 + \alpha\, \Delta T_{AB}) \tag{12}$$

$L_{BC} = (1 + \alpha\, \Delta T_{BC})\sqrt{L^2 + \left(\frac{H}{2}\right)^2}$	(13)
$L_{AC} = (1 + \alpha\, \Delta T_{AC})\sqrt{L^2 + \left(\frac{H}{2}\right)^2}$	(14)

The coordinates of A and B are given by A=[0,0] and B=[0,L_{AB}]. The coordinates of C can then be found by applying Pythagoras' theorem to A, B L_{AB}, L_{BC} and L_{AC}. This gives the effect of thermal expansion as

$$\Delta Cy = \frac{L_{AB}^2 + L_{AC}^2 - L_{BC}^2}{2\, L_{AB}} - \frac{H}{2} \tag{15}$$

$$\Delta Cx = \sqrt{L_{AC}^2 - \frac{L_{AB}^2 + L_{AC}^2 - L_{BC}^2}{2\, L_{AB}}} - L \tag{16}$$

This solution considers the effects of thermal expansion induced shape changes in addition to scaling. Eqn's (10) and (11) consider only scaling. If the shape change solution is considered the best estimate then the errors resulting from applying a scaling based correction are approximated by

$E_{Cx} = \Delta C_{x(scaling)} - \Delta C_x$	(17)
$E_{Cy} = \Delta C_{y(scaling)} - \Delta C_y$	(18)

Substituting through into Eqn's (17) and (18) results in an unwieldly expression from which it is difficult to establish general patterns for the relationship between the input parameters (dimensions of structure, CTE and temperature gradients) and the relative errors. By empirical fitting it is however possible to make some useful generalizations. Firstly for one-dimensional length measurements only E_{Cx} is of relevance and it is negligible for typical length measurements with relatively short lengths and/or high aspect ratios, such as gauge block comparisons. For coordinate measurements E_{cy} is important and is approximated very well by

$$E_{cy} \approx \frac{dT}{dy}\, \alpha\, (A\, H^2 + B\, L^2) \qquad A = 1.25 \text{ x } 10^{-7} \quad B = 5\, x\, 10^{-7} \tag{19}$$

This shows clearly that, for a two-dimensional part, the shape error; 1) Is not affected by the temperature gradient moving away from the datum plane; 2)

Increases linearly with gradient in the perpendicular direction and with the CTE; and 3) Increases quadratically with both length and width but is 4 times more effected by length than by width.

The above example highlights the importance of considering the impact of spatial temperature variation on dimensional uncertainty. For real components, first principles analytical models are unlikely to be practical and Finite Element Analysis (FEA) should be used to determine the sensitivity of critical dimensions to temperature gradients. For structures with spatial variation in the CTE, for example composites and assemblies, the effects of these gradients should also be examined within this modelling process.

Significant shape change should be corrected. Eqn. (19) provides a simple approximation. For more complex part geometry and more accurate corrections FEA should be used. This has been done for machine tools for many years [13] reducing maximum errors by a factor of seven [14]. Applying this technique to correct for errors in measurements is a logical progression [15]. Such approaches, however, lack a rigorous method to determine the uncertainty of these corrections. There are also practical issues in the implementation of these methods which have not been fully resolved in previous work. One major practical issue is obtaining a sufficiently detailed temperature map of the part to make accurate predictions of thermal distortions. In order to calculate the thermal strain within a part an FEA model must be given temperatures measurements at every node in the mesh representing the part. The mesh is likely to involve 10^3 to 10^7 nodes, which limits applicability. Further work is required in this area.

5 Conclusions

It has been shown that the use of higher order terms as recommended by the GUM results in a false sense of an improved estimate of uncertainty. If the difference between a first order and second order solution is significant then this indicates that normality cannot be assumed. In this case MCS should be used to find confidence limits directly. When estimating the uncertainty of a measurement to be carried out in the future worst case estimates of the magnitude of temperature offsets to be corrected are currently used. An improved estimate is obtained by using the uncertainty of this offset although non-normality means that MCS would again provide a better solution. A simple truss model has been used as the basis equations giving an estimate of shape change due to thermal gradients, this effect is important for coordinate measurements and should not be ignored.

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